Exercises 1–2 (5 minutes)

Students work with a partner. Then, discuss and confirm answers as a class.

Exercises 1–2

Suppose that a family has three children. To simulate the genders of the three children, the coin or number cube or a card would need to be used three times, once for each child. For example, three tosses of the coin resulted in HHT, representing a family with two boys and one girl. Note that HTH and THH also represent two boys and one girl.

1. **Suppose that when a prime number (P) is rolled on the number cube, it simulates a boy birth, and a non-prime (N) simulates a girl birth. Using such a number cube, list the outcomes that would simulate a boy birth and those that simulate a girl birth. Are the boy and girl birth outcomes equally likely?**

   The outcomes are 2, 3, 5 for a boy birth and 1, 4, 6 for a girl birth. The boy and girl births are thereby equally likely.

2. **Suppose that one card is drawn from a regular deck of cards. A red card (R) simulates a boy birth, and a black card (B) simulates a girl birth. Describe how a family of three children could be simulated.**

   The key response has to include the drawing of three cards with replacement. If a card is not replaced and the deck shuffled before the next card is drawn, then the probabilities of the genders have changed (ever so slightly, but they are not $50/50$ from draw to draw). Simulating the genders of three children requires three cards to be drawn with replacement.

Example 2 (5 minutes)

This example describes what is meant by a trial, a success, and how to estimate the probability of the desired event (i.e., that a family has three boys or three girls). It uses the coin device, 100 trials, in which 28 of them were either HHH or TTT. Hence, the estimated probability that a family of three children has three boys or three girls is $\frac{28}{100}$.

Example 2

Simulation provides an estimate for the probability that a family of three children would have three boys or three girls by performing three tosses of a fair coin many times. Each sequence of three tosses is called a *trial*. If a trial results in either HHH or TTT, then the trial represents all boys or all girls, which is the event that we are interested in. These trials would be called a *success*. If a trial results in any other order of H's and T's, then it is called a *failure*.

The estimate for the probability that a family has either three boys or three girls based on the simulation is the number of successes divided by the number of trials. Suppose 100 trials are performed, and that in those 100 trials, 28 resulted in either HHH or TTT. Then, the estimated probability that a family of three children has either three boys or three girls would be $\frac{28}{100}$ or 0.28.

- What is the estimated probability that the three children are not all the same gender?
  - $1 - 0.28 = 0.72$

Exercises 3–5 (13 minutes)

Students continue to work with their partners on Exercises 3–5. If time permits after Exercise 3, ask students, "Based on what you found, is it likely that a family with three children will have exactly one girl? From your own experiences, how many families do you know who have three children and exactly one girl?" Discuss the answer to Exercise 5(c) as a class.

Exercises 3–5

3. **Find an estimate of the probability that a family with three children will have exactly one girl using the following outcomes of 50 trials of tossing a fair coin three times per trial. Use H to represent a boy birth and T to represent a girl birth.**

HHT	HTH	HHH	TTH	THT	THT	HTT	HHH	TTH	HHH
HHT	TTT	HHT	TTH	HHH	HTH	THH	TTT	THT	THT
THT	HHH	THH	HTT	HTH	TTT	HTT	HHH	TTH	THT
THH	HHT	TTT	TTH	HTT	THH	HTT	HTH	TTT	HHH
HTH	HTH	THT	TTH	TTT	HHT	HHT	THT	TTT	HTT

T represents a girl. I went through the list, counted the number of times that HHT, HTH, or THH appeared and divided that number of successes by 50. The simulated probability is $\frac{16}{50}$, or 0.32

4. **Perform a simulation of 50 trials by rolling a fair number cube in order to find an estimate of the probability that a family with three children will have exactly one girl.**
   a. **Specify what outcomes of one roll of a fair number cube will represent a boy and what outcomes will represent a girl.**
   b. **Simulate 50 trials, keeping in mind that one trial requires three rolls of the number cube. List the results of your 50 trials.**
   c. **Calculate the estimated probability.**

   Answers will vary. For example, students could identify a girl birth as 1, 2, 3 outcome on one roll of the number cube and roll the number cube three times to simulate three children (one trial). They need to list their 50 trials. Note that an outcome of 412 would represent two girls, 123 would represent three girls, and 366 would represent one girl, as would 636 and 663. Be sure that they are clear about how to do all five steps of the simulation process.

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5. **Calculate the theoretical probability that a family with three children will have exactly one girl.**
   a. **List the possible outcomes for a family with three children. For example, one possible outcome is BBB (all three children are boys).**

   The sample space is BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG.

   b. **Assume that having a boy and having a girl are equally likely. Calculate the theoretical probability that a family with three children will have exactly one girl.**

   Each is equally likely, so the theoretical probability of getting exactly one girl is $\frac{3}{8}$, or 0.375 (BBG, BGB, GBB).

   c. **Compare it to the estimated probabilities found in parts (a) and (b).**

   Answers will vary. The estimated probabilities from the first two parts of this exercise should be around 0.375. If not, suggest that students conduct more trials.